Fast Simulation of Spontaneous Parametric Down-Conversion via Neural Operators

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Abstract: We present a learning approach to simulating Spontaneous Parametric Down-Conversion. Based on Fourier Neural Operators, the method is both fast and resolution independent. The learned operator is able to generalize well, successfully predicting physical observables.

Goals Spontaneous Parametric Down-Conversion (SPDC) is a quantum phenomenon in nonlinear optics that occurs when a single photon of higher energy spontaneously splits into two lower energy photons. More specifically, the high-energy (pump) photon interacts with a nonlinear crystal; due to the crystal's nonlinear properties, the high energy photon is transformed into two lower energy photons, known as "signal" and "idler" photons. SPDC has several important applications in quantum optics and quantum information, including quantum cryptography, quantum metrology, and quantum communication. The dynamics of SPDC may be described by two pairs of nonlinear coupled PDE that do not admit an analytical solution [1]. A standard way to solve such PDEs involves a pseudo-spectral numerical method, the Split-Step Fourier Method (SSFM). Our goal is to design an alternative to SSFM, which is both faster to compute and independent of the resolution of the discretization.

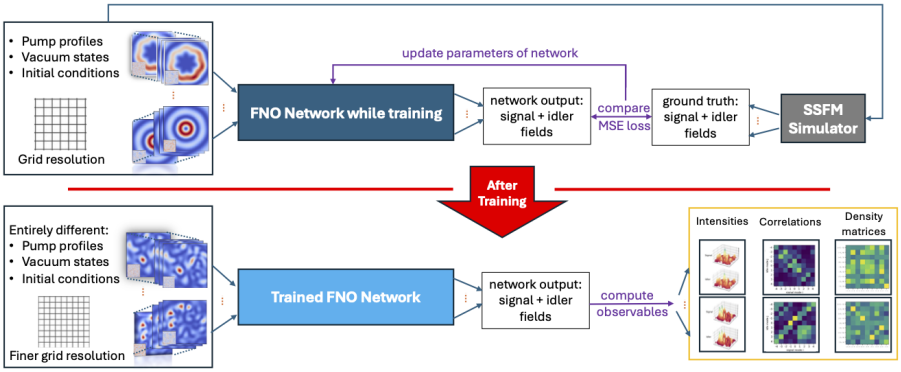

Fig. 1: Schematic overview of the method. Each of the stacks of images on the left represent a given pump with varying vacuum states. The MSE loss is between individual samples (not averaged over vacuum states).

FNO Approach to SPDC Our approach to solving SPDC dynamics relies on a supervised deep learning method. We aim to learn an architecture which takes as input both the initial conditions (IC) of the PDE and the parameters of the problem - both the nonlinear crystal structure and laser pump profile; and whose output is the PDE's solution. The training data is generated as (input, output) pairs, where the input consists of many different ICs and pumps; and the output is provided by SSFM-simulated solutions for that IC-pump configuration. To demonstrate the efficacy of the method, we have fixed the crystal structure. We are particularly interested in measuring the quality of our solution in terms of how well it approximates quantum state observables, the first- and second-order correlations between the idler and signal photons as well as the density matrix of the resultant state at the end of the crystal (which can be computed using the methods introduced in [1, 2]).

More specifically, as detailed in [2], the SPDC dynamics is described by four fields $E = (E_i^{out}, E_i^{vac}, E_s^{out}, E_s^{vac})$ whose interaction is given by a pair of coupled PDE of the form $i\frac{\partial E}{\partial z} = L(\Lambda)E$. In particular, $L(\Lambda)$ is the differential operator dependant on the parameters $\Lambda = (E_p, \chi^{(2)})$, where E_p is the pump profile and $\chi^{(2)}$ the second-order susceptibility describing the crystal structure; both are functions of (x, y, z), with z the propagation direction. We use SSFM to solve for the dynamics of E_i^{out}, E_s^{out} (idler and signal) with E^{vac} (vacuum) as a given and regarded as part of the IC, and this provides us with our training data. We similarly generate test data using an entirely separate set of pump profiles and ICs.

The deep learning approach is based on Fourier Neural Operators (FNO) [3]. FNO is a special case of neural operators (NO), a generalization of neural networks which approximate maps between infinite-dimensional function spaces. The NO architecture is invariant to discretization of the function spaces mapped, since it shares the model parameters for different discretizations of the underlying function spaces [3]. This is a strength of NO-based methods; other learned PDE-solvers, which rely on fixed discretizations, tend to learn undesired grid-based artifacts. In particular, since the model is invariant to discretization, it is able to produce solutions for data in a higher resolution than it was trained on, both in $x - y$ as well as z (different cuts inside the crystal or longer crystals). Finally, another advantage is related to run-time efficiency. The NO's complexity depends solely on the depth of the network, rather than the length of the crystal or its discretization characteristics; contrast this with SSFM which

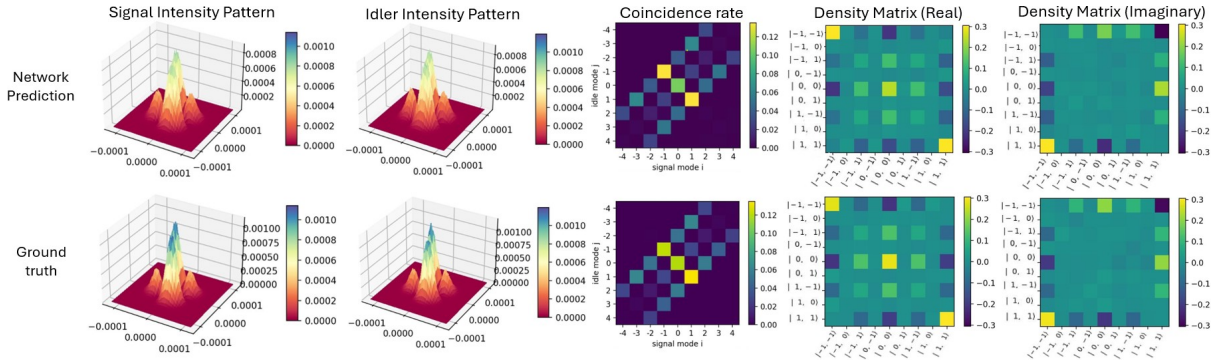

Fig. 2: Network's prediction of LG qutrit-state observables (top row) against ground-truth (bottom row).

depends explicitly on both the latter. The implication is that the FNO-based architecture can potentially solve the SPDC equations considerably faster, most especially for longer crystals.

Results The network was trained on a dataset comprised of distinct random pumps, with a 1mm-long periodically-poled KTP nonlinear photonic crystal. Each sample, initially of dimensions $121 \times 121 \times 10$, underwent random subsampling to a resolution of $30 \times 30 \times 10$ during training. Testing involved 10 diverse random pumps and ICs, entirely independent of the train data.

To evaluate the network's efficacy we compute the quantum state observables [1, 2] and measure the Mean Squared Error (MSE) between observables from the SSFM solution and the network's predictions. This yielded the following average MSE results: signal density distribution: $(1.2 \pm 0.3) * 10^{-10}$, idler density distribution: $(1.6 \pm 0.3) * 10^{-10}$, coincidence rate count: $(1.8 \pm 1.2) * 10^{-5}$, density matrix: $(1.2 \pm 1.1) * 10^{-3}$. The network's performance in predicting a qutrit state is depicted in Figure 2, as experimentally reported by Kovlakov et al. [4]. Additionally, the network was tested for crystal lengths beyond that used during training (up to 2 times the original crystal length) for LG beam $(p = 0, l = 1)$. Results are presented in Table 1, showcasing expected error growth with increasing length, while demonstrating high quality predictive accuracy for lengths unencountered during training; All without incurring any additional runtime overhead.

Regarding time efficiency, a performance comparison was conducted on an Nvidia RTX A4000 GPU with 16 GB RAM. Disabling parallel computation, the classical solver took 368.9 seconds for 1000 samples (0.3689 seconds per sample), while the FNO solver finished in 94.8 seconds (0.0948 seconds per sample); demonstrating a nearly 4 times speedup compared to the classical solver.

$\frac{\textit{Crystal length}}{\textit{Original crystal length}}$	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
signal intensity $[10^{-11}]$	6.93	7.68	8.05	8.46	8.86	9.27	9.70	10.15	10.61	11.08	11.57
idler intensity $[10^{-11}]$	4.96	5.64	5.87	6.08	6.27	6.46	6.65	6.82	7.01	7.18	7.37
coincidence rate count $[10^{-5}]$	4.76	4.51	5.75	7.15	8.62	9.96	11.95	12.34	13.44	14.35	15.40

Table 1: Observables MSE between ground truth and network prediction for different unseen crystal lengths

Discussion We have shown the use of FNO for solving SPDC dynamics. The network achieves comparable accuracy to SSFM in about a quarter of the time. The physical soundness of the solution is shown through accurate predictions of physical observables of the simulated quantum states. The model is invariant to discretization and shows noteworthy generalizability: promising results are shown for unseen laser pump profiles and varying crystal lengths. Future work will focus on generalization to new crystal structures and quantum state engineering. These results hint at the potential of the method to advance applications in quantum communication and computing.

References

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